Math 308: Bridge to Advanced Mathematics
Problem Set 1, due in class at 3:30pm on Tuesday, September 10.

Work on these problems and write down your thoughts, **even if you do not have a complete solution.** **Write clearly** enough for another student in this course, or for yourself in a year, to understand your work.

1. Solve problem 1.1 in our book.
2. Solve problem 1.2 in our book.
3. Solve problem 1.3 in our book.
4. Solve problem 1.10 in our book.
5. Prove 3 of the following statements from the 12 axioms for integers given below.
   (a) $1 \neq 0$
   (b) $1 \not< 0$
   (c) For any integers a, b, if $a \cdot b = 0$, then $a = 0$ or $b = 0$.
   (d) For any integers a, b, c, d, if $a < b$ and $c < d$, then $a + c < b + d$.
   (e) For any integer a, one of the following holds: $a < 0$ or $1 = a$ or $1 < a$.

What we can assume about addition, multiplication, and ordering of integers.

The following are true for all integers a, b, c.

1. $a + b = b + a$ (commutative law of addition).
2. $a + (b + c) = (a + b) + c$ (associative law of addition).
3. $0 + a = a$ (0 is the additive identity).
4. For all a there exists some $\hat{a}$ with the property that $a + (\hat{a}) = 0$ (existence of additive inverses).
5. $a \cdot b = b \cdot a$ (commutative law of multiplication).
6. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ (associative law of multiplication).
7. $1 \cdot a = a$ (1 is the multiplicative identity).
8. $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$ (distributive law).
9. One and only one of the following is true: $a < b$, $a = b$, $b < a$ (total ordering property).
10. If $a < b$ and $b < c$ then $a < c$ (order is transitive).
11. If $a < b$ then $a + c < b + c$ (translation invariance).
12. If $a < b$ and $0 < c$ then $c \cdot a < c \cdot b$ (multiplicative invariance).

These are from pp. 73-74 (78-79 in the pdf) of our main textbook.