Math 308: Bridge to Advanced Mathematics
Problem Set 2, due in class at 3:30pm on Tuesday, September 17.

Work on these problems and write down your thoughts, **even if you do not have a complete solution. Write clearly** enough for another student in this course, or for yourself in a year, to understand your work.

1. Solve problem 1.4 in our book.

2. This question uses the definitions in problem 1.4 in our book. For each of the following, decide whether the statement is true, and prove your answer.

   (a) $\max(a+b, c+d) = \max(a,b) + \max(c,d)$ for any real numbers a, b, c, d.

   (b) $\max(-a, -b) = -(\max(a,b))$ for any real numbers a, b.

   Find and prove correct version(s) of the statement(s) you disproved.

3. (Recall problem 1 from Preview problems, for context.)
   Suppose that L, M, P, and R are reduced; and N, R, S, and W are not reduced.
   Use truth tables as necessary to determine which of the following conditions are satisfied.

   (a) Both M and R are reduced.

   (b) If both G and S are reduced, W is also reduced.

   (c) If N is reduced, neither R nor S is reduced.

   (d) If P is reduced, L is not reduced.

   (e) Of the three areas L, M, and R, exactly two are reduced.

   Find two solutions that satisfy all five of these conditions.

4. For each of the following statements, decide whether or not it is true. Give explicit counterexamples for the false one, and prove the true ones.

   (a) If a and b are rational numbers, then $a+b$ is a rational number.

   (b) If a is a rational number and b is an irrational number, then $a+b$ is a rational number.

   (c) If a is a rational number and b is an irrational number, then $a+b$ is an irrational number.

   (d) If a and b are irrational numbers, then $a+b$ is an irrational number.

   (e) If a and b are irrational numbers, then ab is an irrational number.

   (f) If a and b are irrational numbers, then a^b is an irrational number. Hint: consider $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$.