

Math 308: Bridge to Advanced Mathematics
Preview Problems, due in class at 6:30pm on Thursday, January 31.

Work on these problems and write down your thoughts, **even if you do not have a complete solution**. Write clearly enough for another student in this course, or for yourself in a year, to understand your work.

1. (from <http://www.lsac.org/jd/lsat/prep/analytical-reasoning>)

A university library budget committee must reduce exactly five of eight areas of expenditure G, L, M, N, P, R, S, and W in accordance with the following conditions:

- (a) Both M and R are reduced.
- (b) If both G and S are reduced, W is also reduced.
- (c) If N is reduced, neither R nor S is reduced.
- (d) If P is reduced, L is not reduced.
- (e) Of the three areas L, M, and R, exactly two are reduced.

Which one of the following is a pair of areas neither of which could be reduced?

(G, L) (G, N) (L, N) (L, P) (P, S)

2. For each of the following statements, decide whether or not it is true and write down a clear and convincing explanation for your decision. Some of these are harder than others, and one of them is not known to anyone.

- (a) For every positive integer N , the sum of the cubes of the first N integers is equal to the square of the sum of the first N integers.
- (b) For all primes p , the number $2^p - 1$ is prime.
- (c) There are non-zero integers m and n such that $m(e + \pi) = n$.
(Here, π is the number for which $\cos(\pi) = -1$, and e is the number such that $f(x) = e^x$ is its own derivative.)
- (d) Every continuous function is differentiable.
- (e) Every differentiable function is continuous.
- (f) If a function is differentiable, then its derivative is a continuous function.

General homework directions:

You may discuss homework problems with other students, and I encourage you to do so. However, write your solutions yourself: do not copy them word for word. Acknowledge your collaborators: write on the solutions you hand in "I worked with Jane Lee on problem 3, and with Jose Perez on problems 2a and 4."

If you do not know some of the words used on the homework, look them up before working on the problem: start with our textbooks, then try a dictionary or google. You may also use outside sources, such as books or websites, if you are stuck on a homework problem after at least 30 minutes of thought over at least two days. In any case, provide a traceable reference to your source(s); "wikipedia" or "a theorem in a number theory book" is not traceable; "the wikipedia page for `Equivalence_relation`" and "Theorem 3.7 on p.54 of Burton's Elementary Number Theory" are traceable.

Failing to acknowledge collaboration or outside sources is called plagiarism; it is a kind of cheating. Cheating is taken very seriously in US colleges. **If I find plagiarism in your problem sets, you will receive no credit and no feedback on problem sets for the rest of the semester.**