Math 308: Bridge to Advanced Mathematics
Problem Set 3, due in class on Thursday, February 21st.

Work on these problems and write down your thoughts, **even if you do not have a complete solution. Write clearly** enough for another student in this course, or for yourself in a year, to understand your work.

1. For each of the following statements, decide whether or not it is true. Give explicit counterexamples for the false one, and prove the true ones.

   (a) If a and b are rational numbers, then $a + b$ is a rational number.

   (b) If a and b are irrational numbers, then ab is an irrational number.

   (c) If a is a rational number and b is an irrational number, then $a - b$ is an irrational number.

   (d) If a and b are irrational numbers, then a^b is an irrational number. Hint: consider $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$.

   Use induction to prove each of the following statements.

2. For every positive integer N, the sum of the cubes of the first N integers is equal to the square of the sum of the first N integers. If you use any summation formulas, include their proofs.

3. The sum of the cubes of any three consecutive positive integers is an integer multiple of 9.

4. The number $n^3 + 5n$ is divisible by 3 for every positive integer n.

5. Use trigonometric identities to show that for any positive integer n, there are polynomials p_n and q_n with integer coefficients such that for any real x,

$$\cos(nx) = p_n(\cos(x)) \quad \text{and} \quad \sin(nx) = \sin(x) \cdot q_n(\cos(x)).$$

General homework directions:

You may discuss homework problems with other students, and I encourage you to do so. However, write your solutions yourself: do not copy them word for word. Acknowledge your collaborators: write on the solutions you hand in "I worked with Jane Lee on problem 3, and with Jose Perez on problems 2a and 4."

If you do not know some of the words used on the homework, look them up before working on the problem: start with our textbooks, then try a dictionary or google. You may also use outside sources, such as books or websites, if you are stuck on a homework problem after at least 30 minutes of thought over at least two days. In any case, provide a traceable reference to your source(s); "wikipedia" or "a theorem in a number theory book" is not traceable; "the wikipedia page for `Equivalence_relation`" and "Theorem 3.7 on p.54 of Burton's Elementary Number Theory" are traceable.

Failing to acknowledge collaboration or outside sources is called plagiarism; it is a kind of cheating. Cheating is taken very seriously in US colleges. **If I find plagiarism in your problem sets, you will receive no credit and no feedback on problem sets for the rest of the semester.**