Math 308: Bridge to Advanced Mathematics
Set notation, due in class on Tuesday, April 9.

Work on these problems and write down your thoughts, **even if you do not have a complete solution.** **Write clearly** enough for another student in this course, or for yourself in a year, to understand your work.

1. Playing with changing the order of set operations.
   (a) Let $A := \{7, 8\}$. Evaluate $A \times ([(A \cup A) \cup A] \times A)$ and $(A \times A) \cup [(A \cup A) \times A]$.
   (b) Let $A := \{\heartsuit\}$. Evaluate $A \times ([(A \cup A) \cup A] \times A)$ and $(A \times A) \cup [(A \cup A) \times A]$.
   (c) Add parentheses to the expression
   $$A \times A \cup A \cup A \times A$$
   in a new way, so that for $A := \{7, 8\}$ you get a set that is different from the ones in part (a).
   (d) Is there a way to add parentheses to the expression
   $$A \times A \cup A \cup A \times A$$
   to get a new set when $A := \{\heartsuit\}$?
2. Is $\emptyset = \emptyset \times \emptyset$?
3. Let $X := \{a, 3, \star\}$. Write down the eight elements of $\mathcal{P}(X)$.
4. In this problem, x and y are real numbers; and X, S, A, B, and E are sets of real numbers.
   (a) Explain in words the following property of a set X:
   $$\exists x \in X \, \forall y \in X \; x \leq y.$$
   (b) Write down the negation of the property in part (a).
   (c) Suppose that $S \subseteq \mathbb{N}$ has the property in part (b); show that $1 \notin S$.
   (d) Find a set $E \subseteq \mathbb{N}$ that has the property in part (b).
   (e) Use strong induction to prove that any $S \subseteq \mathbb{N}$ that has the property in part (b) is equal to the set E you found in part (d).
   (f) Find sets $B \subseteq A$ such that A has the property in part (a) and B has the property in part (b).
   (g) Use generalized induction to prove that there are no subsets A and B of integeres such that $B \subseteq A$ and A has the property in part (a) and B has the property in part (b).
5. Consider the set
   $$S := \{I \subseteq \mathbb{R} : \forall x \in I \, \forall y \in \mathbb{R} \, \forall z \in I \; \text{ if } x \leq y \leq z, \text{ then } y \in I\}.$$
   (a) Give four examples of sets of real numbers that are elements of S.
   (b) Give four examples of sets of real numbers that are not elements of S.
   (c) What are elements of S called in calculus books?