Math 32404: Advanced Calculus II
Problem set 0, due on Wednesday, August 30th, at 6pm.
Solutions turned in after 4:05pm are late and get half credit.

Your solutions should include explanations that would be understandable and convincing to your classmates. Ideally, your solutions should look like examples and proofs in our textbook.

Please cite any outside sources (books, webpages, experts) that you consult, any technology (calculators, computer software, slide rules) that you use, and any classmates that you collaborate with.

1. Read the first seven sections of our textbook, most of which should be review. Make a list of definitions and theorems in these section that are new to you.

2. Which one the following statements is not equivalent to the other two?

   (a) If a zonk is giggly, then it is dangerous.

   (b) If a zonk is dangerous, then it is giggly.

   (c) If a zonk is not giggly, then it is not dangerous.

   Describe a zonk that would be a counterexample to exactly one of the above three statements.

3. The statement "All yellow numbers are smaller than some blue number" is ambiguous. Find sets $Y \subset \mathbb{R}$ of yellow numbers and $B \subset \mathbb{R}$ of blue numbers making exactly one of the following statements true.

   (a) For every yellow number a there is a blue number b such that $a < b$.

   (b) There is a blue number d such that for every yellow number c, it is true that $c < d$.

4. Recall that for a function f and real numbers a and L, the notation "$\lim_{x\to a} f(x) = L$" means:

   "For every real number $\epsilon \gneq 0$ there exists a real number $\delta \gneq 0$ such that for all real numbers x, if $0 \lneq |x - a| \lneq \delta$, then $|f(x) - L| \leq \epsilon$."

   Evaluate each of the following limits. If the limit exists, find a δ satisfying the above definition for $\epsilon = 0.1$. If the limit does not exist, find an ϵ for which no δ satisfies the above definition.

   (a) $\lim_{t\to 3} 2^t$.

   (b) $\lim_{r\to 0} \frac{1}{r} - \lfloor \frac{1}{r} \rfloor$, where $\lfloor y \rfloor$ is the greatest integer less than or equal to y.

   (c) $\lim_{x\to 1}(x^2 - 1)\sin(\frac{x}{x-1})$.

5. Find all eigenvalues of the matrix $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, and find an eigenvector for each of the eigenvalues. Show work if you do this by hand, cite technology if you use technology.

6. Some of the following sets are graphs of functions. For each of them, either find the function and its domain, or prove that the set is not the graph of a function; and in either case, sketch a graph of the set. Remember to cite technology if you use any!

   (a) $\{(x, y) \in \mathbb{R}^2 : e^y = x^2\}$

   (b) $\{(\cos t, \sin t) : t \in \mathbb{R}\}$

   (c) $\{(x, y) \in \mathbb{R}^2 : x^2 - y^2 = 16\}$

   (d) $\{((\cos t)^2, (\sin t)) : t \in \mathbb{R}\}$