Math 32404: Advanced Calculus II Problem set 3, due on Wednesday, October 25th, at 6pm.

Your solutions should include explanations that would be understandable and convincing to your classmates. Ideally, your solutions should look like examples and proofs in our textbook.

Please cite any outside sources (books, webpages, experts) that you consult, any technology (calculators, computer software, slide rules) that you use, and any classmates that you collaborate with.

- 1. (a) Consider the preimage of the image of a set S under a function f. Show that $f^{-1}(f(S)) = S$, or find a counterexample.
 - (b) Consider the image of the preimage of a set S under a function f. Show that $f(f^{-1}(S)) = S$, or find a counterexample.
- 2. Fill in each of two blanks in the following statement with one of the words provided to make the statement true; and prove the statement.

"If f is a continuous function and A is a [] subset of the domain of
f , then $f^{-1}(f(A))$ is a	[] set."

Words: open, closed, bounded, connected, compact. Find counterexamples to the other 24 ways of filling in the blanks.

- 3. Investigate the relation between between m in Corollary 22.8 and the norm defined in exercises 21.L and 21.M with some simple examples.
- 4. Find a function of two variables that is continuous in each variable alone at all points, but discontinuous as a function of two variables, also at all points; or prove that this is impossible.
- 5. Give an example of a sequence of continuous functions which converges to a continuous function, but where the convergence is not uniform.
- 6. Suppose the $S \subset \mathbb{R}^2$ is a connected set containing the points (17, -17) and (-17, 17). Use the intermediate value theorem to prove that S must contain a point on the line x = y; or find a counterexample.