Math 32404: Advanced Calculus II
Problem set 5, due on Monday, November 27th, at 6pm.
Solutions turned in after 4:05pm are late and get half credit.

Your solutions should include explanations that would be understandable and convincing to your classmates. Ideally, your solutions should look like examples and proofs in our textbook.

Please cite any outside sources (books, webpages, experts) that you consult, any technology (calculators, computer software, slide rules) that you use, and any classmates that you collaborate with.

1. Let $f(x, y) := e^{x+y}$.
   (a) Compute the two partial derivatives of f of order 1.
   (b) Compute all the partial derivatives of f of all orders.
   (c) Compute the third derivative $(D^3 f(1,1)) : \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$.
2. (a) Compute the two first partial derivatives of $e^{x+y}(x-y)^2$ at $(1,1)$.
   (b) Compute the first and second partial derivatives of $e^{x+y}(x-y)^3$ at $(1,1)$.
   (c) Consider the function $g(x, y) := e^{x+y}(x-y)^n$ for some integer $n > 1$. Generalize your observations from parts (a) and (b) about the partial derivatives of g at $(1,1)$ and prove them.
3. Let
$$f(x,y) := \begin{cases} \frac{y(x-y)(x+y)}{x} & \text{if } |y| \lneq |x|, \\ 0 & \text{otherwise.} \end{cases}$$
   (a) Compute $D_x f$ at points $(0, b)$ on the y-axis; and $D_y f$ at points $(a, 0)$ on the x-axis.
   (b) Compute $D_{xy} f$ and $D_{yx} f$ at $(0,0)$.
   (c) From (b), this function does not satisfy the conclusion of Theorem 40.8. Which hypotheses of that theorem does it fail to satisfy? Which steps in the proof of that theorem fail?
   (d) Repeat parts (a)-(c) for
$$g(x,y) := \begin{cases} \frac{y(x-y)^2(x+y)^2}{x^3} & \text{if } |y| \lneq |x|, \\ 0 & \text{otherwise.} \end{cases}$$
   (e) Repeat parts (a)-(c) for
$$h(x,y) := \begin{cases} \frac{y(x-y)^3(x+y)^3}{x^5} & \text{if } |y| \lneq |x|, \\ 0 & \text{otherwise.} \end{cases}$$
4. Use Chain Rule to prove that whenever a function $f : \mathbb{R}^p \to \mathbb{R}$ is differentiable at a point $a \in \mathbb{R}^p$ such that $f(a) \neq 0$, the function $g := 1/f$ is differentiable at a.
5. Solve exercises 40.J and 40.K in our text.