Math 32404: Advanced Calculus II
Reading exercises 1, due on Monday, September 6th.

Read sections 8-13 in our textbook, including the exercises in the textbook, and solve the exercises below as you go along. Your solutions will not be collected, but a very short in-class quiz on the due date will contain one of these exercises, or one very very similar to it.

1. Compute $(1, 2, 3) \cdot (2, 6, 1)$.
2. Compute $\|(1, -2, 3)\|$ and $\|(1, 2, 3)\|_1$ and $\|(1, 2, 3)\|_\infty$.
3. Describe the set of all vectors in $\mathbb{R}^3$ that are orthogonal to the vector $(0, 0, 1)$.
4. Describe the set of all vectors in $\mathbb{R}^3$ that are orthogonal to the vector $(0, 1, 1)$.
5. For each of the following, decide whether it is convex, open, closed, compact, connected, bounded.
    (a) The open ball of radius 3 with center $(1, 2, 3)$.
    (b) The closed ball of radius 1 with center $(1, 2, 3, 4)$.
    (c) The sphere of radius 5 with center $(1, 2)$.
    (d) The sphere of radius 5 with center (1).
    (e) The answer to exercise 3.
    (f) The answer to exercise 4.
    (g) The empty set.
    (h) $\mathbb{R}^{17}$
    (i) The open cell $\{(x_1, x_2, x_3) \in \mathbb{R}^3 : \pi < x_1 < \pi^2 + 2,\ 3 < x_2 < 5,\ -5 < x_3 < 0\}$
6. Find a subset of $\mathbb{R}$ that is not open, not closed, not convex, not compact, not bounded, and not connected.
7. Find a collection of open subsets of $\mathbb{R}^3$ whose intersection is not open.
8. Find a collection of closed subsets of $\mathbb{R}^3$ whose union is not closed.
9. Let $A := \{(x, y) \in \mathbb{R}^2 : \|(x, y)\|_1 \leq 2\}$. Which points in $\mathbb{R}^2$ are interior points of A? exterior points of A? boundary points of A?
10. Let $A := \{(x, y) \in \mathbb{R}^2 : \|(x, y)\|_1 \geq 2\}$. Which points in $\mathbb{R}^2$ are interior points of A? exterior points of A? boundary points of A?
11. Describe the interior A° of the closed cell
$$A := \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 1 \leq x_1 \leq 2,\ 3 \leq x_2 \leq 7,\ -3 \leq x_3 \leq 0\}.$$
12. Let A be a subset of $\mathbb{R}^p$; let A° be the set of all interior points of A, let B be the set of all boundary points of A, let E be the set of all exterior points of A, and let C be the set of all cluster points of A. Consider the 16 possible intersections of A°, B, E, C and their complements, such as $A^\circ \cap B \cap (\mathbb{R}^p \setminus E) \cap (\mathbb{R}^p \setminus C)$. Which 4 of these 16 might be nonempty?