Math 32404: Advanced Calculus II
Reading exercises 4, due on Monday, October 30th.

Read section 39 in our textbook, including the exercises in the textbook, and solve the exercises below as you go along. **In addition, review single-variable sections 27-28 as necessary.** Your solutions will not be collected, but a very short in-class quiz on the due date will contain one of these exercises, or one very very similar to it.

1. Let $f(r, s, t) := (r \cos s \sin t, r \sin s \sin t, r \cos t)$.
   (a) Compute the 9 partial derivatives of f.
   (b) Evaluate $f(2, 0, \pi/2)$ and the Jacobian matrix representing the derivative of f at $(2, 0, \pi/2)$.
   (c) Find some $\delta > 0$ that satisfies the definition of "f is differentiable at $(2, 0, \pi/2)$" for $\epsilon = .01$.
   (d) At what points in $\mathbb{R}^3$ is the Jacobian determinant of f non-zero?
2. Write out the proof of Theorem 39.9 in full generality, that is, without assuming that $q = 1$. Exercise 39.V may be helpful - but prove it before using it!
3. Solve at least two of exercises 39.A - H.
4. Compute at least one item from each of the exercises 39.O, 39.Q, and 39.S.
5. Verify at least one of the exercises 39.N, 39.P, and 39.R.
6. (*)Find proofs of all parts of Theorem 27.9 in books, or write down your own.
7. (*)Let $f_n(x) := x^n$ for each $n = 1, 2, 3, \ldots$. For what intervals $J \subset \mathbb{R}$ does Theorem 28.5 apply to this sequence of functions? Compute everything mentioned in the theorem and its proof for this sequence of functions for the interval $[-0.1, 0.7]$; and also for the interval $[-1.7, 1.7]$.
8. (*)Do one of exercises 28.K and 28.L.

(*) These will not appear on the quiz, problem set, or test. However, our next topic is multivariable versions of these.