

Math 32404: Advanced Calculus II
Problem set 1, due on Tuesday, February 13th, at 4pm.
Solutions turned in after 4:05pm are late and get half credit.

Your solutions should include explanations that would be understandable and convincing to your classmates. Ideally, your solutions should look like examples and proofs in our textbook.

Please cite any outside sources (books, webpages, experts) that you consult, any technology (calculators, computer software, slide rules) that you use, and any classmates that you collaborate with.

1. Let's call real numbers whose decimal expansion can be written using only the digits 3 and 7 *three-sevenny*. So, for example, $\frac{1}{3}$ and $\frac{7}{9}$ are three-sevenny, but $\frac{5}{2}$ and $\frac{3}{5}$ are not. Let $X \subset \mathbb{R}^2$ be the set of points both of whose coordinates are three-sevenny. Is X open? closed? connected?
2. For what n is it true that any connected, unbounded subset of \mathbb{R}^n must be either open or closed? Prove the statement for those n for which it is true, and find counterexamples for all other n .
3. Fix n and $A \subset \mathbb{R}^n$; let a be an interior point of A , and let e be an exterior point of A . Prove that the line segment with endpoints a and e contains a boundary point of A ; or find a counterexample.
4. Prove that for any n and any two open sets $U, V \subset \mathbb{R}^n$, if $U \cap \mathbb{Q}^n = V \cap \mathbb{Q}^n$, then $U = V$; or find a counterexample.
5. Solve exercises 11.N - 11.Q on p. 80 of our textbook. You may use 11.N for the later ones even if you do not solve it.