Math 32404: Advanced Calculus II
Problem set 2, due on Wednesday, October 4th, at 6pm.
Solutions turned in after 4:05pm are late and get half credit.

Your solutions should include explanations that would be understandable and convincing to your classmates. Ideally, your solutions should look like examples and proofs in our textbook.

Please cite any outside sources (books, webpages, experts) that you consult, any technology (calculators, computer software, slide rules) that you use, and any classmates that you collaborate with.

1. Let $s_0 := (1, 2) \in \mathbb{R}^2$, and let $s_{n+1} := s_n + 2^{-n}(\cos(n), \sin(n))$. Does this sequence converge? Prove your answer!
2. For a sequence (s_n) in $\mathbb{R}^p$, let $d_n := \|s_n - s_{n+1}\|$. Prove that the sequence (s_n) converges if and only if the sequence (d_n) converges to 0; or find a counterexample.
3. Suppose that (s_n) and (t_n) are two sequences in $\mathbb{R}^2$, and define another sequence (u_n) by $u_{2m} := s_m$ and $u_{2m+1} := t_m$.
   (a) Prove that (u_n) converges whenever both (s_n) and (t_n) converge; or find a counterexample.
   (b) Prove that (u_n) converges to u whenever both (s_n) and (t_n) converge to u; or find a counterexample.
4. Let (f_n) be a sequence of functions from $\mathbb{R}^p$ to $\mathbb{R}^q$, and let $D_1 \subset D_2 \subset \ldots \subset \mathbb{R}^p$ be a countable collection of open subsets D_i of $\mathbb{R}^p$ with $D_i \subset D_{i+1}$ for all i; and let $D := \cup_i D_i$.
   (a) Suppose that the sequence (f_n) converges on each D_i. Prove that (f_n) converges on D; or fnd a counterexample.
   (b) Suppose that the sequence (f_n) converges uniformly on each D_i. Prove that (f_n) converges uniformly on D; or fnd a counterexample.
5. Let (f_n) be a convergent sequence of functions such that $f_n \in B_{pq}(D)$ for all n. Prove that the norm $\|f\|_D$ of the limit f of this sequence is equal to the limit of the sequence $s_n := \|f_n\|_D$ of norms of the functions in the sequence; or find a counterexample.
6. Suppose that (f_n) is a convergent sequence of constant functions on D. Prove that the limit function f is also constant; or find a counterexample.
7. Suppose that (f_n) is a convergent sequence of differentiable functions on $[1, 2] \subset \mathbb{R}^1$. Prove that the limit function f is also differentiable; or find a counterexample.
8. Suppose that (f_n) is a convergent sequence of polynomials on D. Prove that the limit function f is also a polynomial; or find a counterexample.