Math 32404: Advanced Calculus II
Reading exercises 1, due on Thursday, February 8th.

Read sections 8-13 in our textbook, including the exercises in the textbook, and solve the exercises below as you go along. Your solutions will not be collected, but a very short in-class quiz on the due date will contain one of these exercises, or one very very similar to it.

1. Compute $(1,2,3)\cdot(2,6,1)$.
2. Compute $\|(1,-2,3)\|$ and $\|(1,2,3)\|_1$ and $\|(1,2,3)\|_\infty$.
3. Describe the set of all vectors in $\mathbb{R}^3$ that are orthogonal to the vector $(0,0,1)$.
4. Describe the set of all vectors in $\mathbb{R}^3$ that are orthogonal to the vector $(0,1,1)$.
5. For each of the following, decide whether it is convex, open, closed, compact, connected, bounded.
   (a) The open ball of radius 3 with center $(1,2,3)$.
   (b) The closed ball of radius 1 with center $(1,2,3,4)$.
   (c) The sphere of radius 5 with center $(1,2)$.
   (d) The sphere of radius 5 with center (1).
   (e) The answer to exercise 3.
   (f) The answer to exercise 4.
   (g) The empty set.
   (h) $\mathbb{R}^{17}$
   (i) The open cell $\{(x_1,x_2,x_3)\in\mathbb{R}^3 : \pi < x_1 < \pi^2+2,\ 3 < x_2 < 5,\ -5 < x_3 < 0\}$
6. Find a subset of $\mathbb{R}$ that is not open, not closed, not convex, not compact, not bounded, and not connected.
7. Find a collection of open subsets of $\mathbb{R}^3$ whose intersection is not open.
8. Find a collection of closed subsets of $\mathbb{R}^3$ whose union is not closed.
9. Let $A := \{(x,y)\in\mathbb{R}^2 : \|(x,y)\|_1 \le 2\}$. Which points in $\mathbb{R}^2$ are interior points of A? exterior points of A? boundary points of A?
10. Let $A := \{(x,y)\in\mathbb{R}^2 : \|(x,y)\|_1 \ge 2\}$. Which points in $\mathbb{R}^2$ are interior points of A? exterior points of A? boundary points of A?
11. Describe the interior A° of the closed cell
$$A := \{(x_1,x_2,x_3)\in\mathbb{R}^3 : 1\le x_1\le 2,\ 3\le x_2\le 7,\ -3\le x_3\le 0\}.$$
12. Let A be a subset of $\mathbb{R}^p$; let A° be the set of all interior points of A, let B be the set of all boundary points of A, let E be the set of all exterior points of A, and let C be the set of all cluster points of A. Consider the 16 possible intersections of A°, B, E, C and their complements, such as $A^\circ\cap B\cap(\mathbb{R}^p\setminus E)\cap(\mathbb{R}^p\setminus C)$. Which 4 of these 16 might be nonempty?