Math 32404: Advanced Calculus II
Reading exercises 2, due on Thursday, February 15th.

Read sections 14-17 in our textbook, including the exercises in the textbook, and solve the exercises below as you go along. Your solutions will not be collected, but a very short in-class quiz on the due date will contain one of these exercises, or one very very similar to it.

1. For what real numbers a does the sequence $s_n := a^n$ converge? (Try $a = 0, 1 - 1, 2, -2, 1/2, -1/2$ if you are not sure how to start.)
2. For what real numbers a is the sequence $s_n := a^n$ monotone?
3. For what real numbers a is the sequence $s_n := a^n$ bounded?
4. For what real number a does the sequence $s_n := a^n$ have a convergent subsequence even though it does not converge? What are the limits of its convergent subsequences?
5. For what real numbers a does the sequence $t_n := (\sin an, \cos an)$ converge?
6. For what real numbers a is the sequence $t_n := (\sin an, \cos an)$ bounded?
7. For what real numbers a does the sequence $t_n := (\sin an, \cos an)$ have a convergent subsequence even though it does not converge? What are the limits of these convergent subsequences?
8. Fix p, let A be a subset of $\mathbb{R}^p$, and let (u_n) be a convergent sequence in $\mathbb{R}^n$ such that $u_n \in A$ for every n. Prove that the limit u of this sequence is an interior point of A or a boundary point of A.
9. Fix p, let A be a closed subset of $\mathbb{R}^p$, and let (u_n) be a convergent sequence in $\mathbb{R}^n$ such that $u_n \in A$ for every n. Prove that the limit u of this sequence lies in A.
10. Fix p, let A be a compact subset of $\mathbb{R}^p$, and let (u_n) be a sequence in $\mathbb{R}^n$ such that $u_n \in A$ for every n. Prove that there exists a convergent subsequence of (u_n) whose limit lies in A.
11. Note that the set V_p of all sequences in $\mathbb{R}^p$ is a vector space. Find an infinite linearly independent subset of V_2.
12. Find a sequence of functions (g_n) from $\mathbb{R}^2$ to $\mathbb{R}^2$ which converges to a constant function on all of $\mathbb{R}^2$, but is not uniformly convergent on $\mathbb{R}^2$.