Math 32404: Advanced Calculus II Reading exercises 6, due on Tuesday, May 1st.

Read pages 375-387 of section 41 in our textbook with the examples provided below. This section is significantly more difficult than previous ones.

Your solutions will not be collected, but a very short in-class quiz on the due date will contain one of these exercises.

* This linear algebra review question will not appear on the reading quiz. Let

$$M := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix},$$

and let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be the affine function given by L(x) = Mx + (1, 1, 1).

- (a) Find the matrix representing a linear function whose graph is a *level* set of L.
- (b) Find the matrix representing a linear function one of whose *level sets* is the *graph* of *L*.
- (c) Find an affine function whose *image* is the graph of L.
- (d) Find an affine function whose graph is the *image* of L.
- 1. Read pp. 375-382 under the assumption that p = q = 1. Work through the details of the proofs with the following examples:

$$f(x) = x^3$$
, $f(x) = x^2$, $f(x) = x|x| + x$.

- 2. Read the same pages again, this time with p = 1 and q = 2. Which of 4.1-4.8 do not apply to this case? Work through the rest of 41.1-41.8 with $f(x) = (x^2, x^3)$ and with $f(x) = (x^2 1, x^3 x)$. Sketch the graphs of these functions (in \mathbb{R}^3) and their images (in \mathbb{R}^2).
- 3. Read the same pages again, this time with p = 2 and q = 1. Which of 4.1-4.8 do not apply to this case? Work through the rest of 41.1-41.8 with $f(x, y) = \sin(x + y)$. Sketch the graph (in \mathbb{R}^3) of this function, and its image (in \mathbb{R}^2).
- 4. Read the same pages again, this time with p = 2 = q and $f(r,t) = (r \cos t, r \sin t)$.
- 5. Read "Implicit Functions" on pp. 382-387 and work through 41.9 and 41.10 with the following:
 - (a) p = q = 1 and $F(x, y) = x^2 + 4y^2 9$;
 - (b) p = q = 1 and F(x, y) = xy + x + y + 1;
 - (c) p = q = 1 and $F(x, y) = x^3 y^2 + c$ for c = -1, 0, 1;

- (d) p = 2 and q = 1 and F(x, y, z) = xyz;
- (e) p = 2 and q = 1 and $F(x, y, z) = x^2 + y^2 + z^2$; (f) p = 1 and q = 2 and $F(x, y, z) = (xyz, x^2 + y^2 + z^2)$.