Math A4400: Mathematical Logic
0th homework set, due at 2pm on monday, september 9th.
Bring your solutions class, or slide them under the door of my office NAC 6278.

1. (Here, I make sure that you know what a proof is.)
   Write a careful, complete, coherent proof of each of the following statements. Write complete sentences; include all details; start by explaining what the statement means.

   (a) There are infinitely many primes.

   (b) The sum of the cubes of the first n positive integers is the square of the sum of these numbers.

2. (Here, you use what you learned in the first lecture.)

   (a) Analyze the following English sentences as we did in lecture and make a truth table for each one. (Start by identifying the basic parts of each statement and picking a sentence symbol for each part.)

       i. The moon is made of green cheese if and only if I am not the princess of Monaco.
       ii. If the moon is made of green cheese and I am the princess of Monaco, then this is one strange world.

   (b) Which row of your truth tables corresponds to reality, to the best of your knowledge? Which (if any) of the two sentences in (a) are true in this row?

   (c) Does one of the two sentences follow from the other?

3. (Here, you think deeply about constructions that take infinitely many steps. These often happen in mathematics.)

   A train with infinitely many seats, one for each rational number, stops in countably many villages, one for each positive integer, in increasing order, and then finally arrives at the city. (For example, the distances between villages could be getting smaller, or the speed of the train faster, so that the train stops at the first village at noon, at the second village at 1pm, and the third at 1:30pm, and the fourth at 1:45pm, and so on until it arrives at the city at 2:17pm.)

   At the first village, two women board the train. At the second village, one woman leaves the train to go visit her cousin, and two other women board the train. At the third village, one woman leaves the train to go visit her cousin, and two other women board the train. At the fourth village, and in fact at every later village, the same thing keeps happening: one woman off to visit her cousin, two new women on board the train. How many women arrive at the city? Explain your answer!