Math A4400: Mathematical Logic

2st problem set, due at 2pm on wednesday, september 25th.

Bring your solutions class, or slide them under the door of my office NAC 6278.

1. (a) Suppose that $s_i \in \mathit{Symb}$ are symbols (that is, sentence symbols or boolean connectives), and suppose that the expression $s_1 s_2 s_3 \dots s_n$ is a sentence. Show that for every $i \leq n$ there is some j such that $i \leq j \leq n$ and $s_i s_{i+1} \dots s_j$ is also a sentence.

   (b) Use (a) and the lemma in the middle of the proof of unique readability to show that the j in (a) is unique.

   (c) Define a function subs : Sent $\to \mathcal{P}$(Sent) recursively as follows:

   $$\mathrm{subs}(p_i) := \{p_i\}, \qquad \mathrm{subs}(\neg\phi) := \mathrm{subs}(\phi) \cup \{\neg\phi\},$$

   and for each binary connective $\star$,

   $$\mathrm{subs}(\star\phi\psi) := \mathrm{subs}(\phi) \cup \mathrm{subs}(\psi) \cup \{\star\phi\psi\}.$$

   Compute $\mathrm{subs}(\wedge \to p_2 \neg p_3 \to p_2 \neg p_3)$.

   (d) Show that the sentence $s_i s_{i+1} \dots s_j$ obtained in (a) belongs to $\mathrm{subs}(s_1 \dots s_n)$.

2. For each set Γ of sentences below, find a truth assignment that satisfies it (it suffices to describe the atomic truth assignment), or find a finite subset of Γ that is not satisfiable.

   (a) $\Gamma := \{\leftrightarrow p_i \neg p_{i+1} \mid i \in \mathbb{N}\}$

   (b) $\Gamma := \{\leftrightarrow \wedge p_i p_j \neg p_k \mid i, j, k \in \mathbb{N}, i \neq j, j \neq k, i \neq k\}$

3. Suppose that $X \subset$ Sent is a set of sentences and there are at least two distinct truth assignments that both satisfy X. Show that there is some satisfiable set of sentences $Y \supsetneq X$.

4. Ask an interesting question about this week's material and try to answer it. This question is as serious as the rest of them!