Math A4400: Mathematical Logic
4th problem set, due at 2pm on wednesday, october 9th.

Bring your solutions class, or slide them under the door of my office NAC 6278.

1. Let S be a signature with one unary relation symbol $+$, a binary function symbol 1, and a 17-ary relation symbol $<$.
   (a) Give and example of an S-structure with infinite universe.
   (b) Give and example of an S-structure with finite universe.

2. Let S_{ring} be the signature of rings: two binary function symbols $\dot{+}$ and $\dot{\times}$, a unary function symbol $\dot{-}$, and two constant symbols $\dot{0}$ and $\dot{1}$. How many S_{ring}-structures with universe $\{1, 2, \clubsuit\}$ are there?

3. For each of the following, prove the statement if it is true, and find a counterexample if it is not.
   (a) If $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ are all S-structures for some signature S, and $\phi : \mathcal{A} \to \mathcal{B}$ and $\psi : \mathcal{B} \to \mathcal{C}$ are S-homomorphisms, then $\psi \circ \phi$ is also an S-homomorphism.
   (b) If $\mathcal{A}$ and $\mathcal{B}$ are S-structures for some signature S, and $\phi : \mathcal{A} \to \mathcal{B}$ is an S-homomorphism and a bijection, then ϕ^{-1} is also an S-homomorphism.

4. Ask an interesting question about this week's material and try to answer it. This question is as serious as the rest of them!