Math A4400: Mathematical Logic
Problem set 0, due at 2pm on thursday, february 4th.
Solutions turned in after 2:05pm are late and get half credit.
You may also bring your solutions in my office NAC 6278; if I am not there, slide them under the door.

Questions labeled with * are somewhat harder.
All page numbers and chapter numbers refer to Mathematical Logic Lecture Notes by van den Dries. Before working on problem 2, read pages 13-16.

1. (Here, I make sure that you are comfortable with basic mathematical notation (summarized in Chapter 1 (Preliminaries)), and that you can write precise coherent proofs.)

   Let A be the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and let 1_A be the identity function from A to itself.

   (a) Write a very detailed proof that for any $m, n \in \mathbb{N}$ and any $x, y \in A$, if $10m + x = 10n + y$, then $x = y$ and $m = n$.

   (b) Write a very detailed proof that there exists a unique function $f : \mathbb{N} \to \mathbb{N}$ extending 1_A and satisfying $f(10m + x) = f(m) + f(x)$ for all $m \in \mathbb{N}$ and all $x \in A$.

   (c) Evaluate $f(2016)$ and $f(20160204)$; how would you describe this function to a 10-year-old?

   * How do these questions (a)-(c) change if x and y can be any natural numbers, rather than elements of A?

2. (Here, you use what you learned in the first lecture.)

   (a) Translate the following two English sentences into propositions.
   (Start by identifying the basic parts of each statement and picking an atom for each part.)

      i. The moon is made of green cheese if and only if I am not the princess of Monaco.

      ii. If the moon is made of green cheese and I am the princess of Monaco, then this is one strange world.

   (b) If you used the symbols $\to$ or $\leftrightarrow$ in part (a), replace them according to the remark on page 15, so using only atoms and the boolean connectives $\wedge$, $\vee$, and $\neg$. Describe how each proposition is obtained by applying the rules (i)-(iii) on the top of page 14.

   (c) Make a truth table for each proposition, like the one on p.15 and like the one we made in lecture.

   (d) What truth assignment on your atoms (i.e. which row of your truth tables) corresponds to reality, to the best of your knowledge? Which (if any) of the two sentences in (a) are true under this truth assignment?

   (e) Is one of the two sentences a tautological consequence of the other?