Math A4400: Mathematical Logic
Problem set 3, due at 2pm on thursday, february 25th.
Solutions turned in after 2:05pm are late and get half credit.

This problem set is about sections 2.1 and 2.2 of Mathematical Logic Lecture Notes by van den Dries.

1. Let $A := \{p_i : i \in \mathbb{N}\}$, and consider the following subsets of $\mathrm{Prop}(A)$. For each set Γ, find a model of Γ or find a finite subset $\Gamma_0 \subset \Gamma$ that is not satisfiable.

   (a) $\Gamma := \{\leftrightarrow p_i \neg p_{i+1} : i \in \mathbb{N}\}$

   (b) $\Gamma := \{\leftrightarrow \wedge p_i p_j \neg p_k : i, j, k \in \mathbb{N}, i \neq j, j \neq k, i \neq k\}$

2. Show that the set Γ_0 that you found in Problem 1 is inconsistent.

3. Let $X \subset \mathrm{Prop}(A)$ be a set of propositions on A. Suppose that X has at least two distinct models. Show that there is some satisfiable set $Y \subset \mathrm{Prop}(A)$ such that $Y \supsetneq X$; or find a counterexample.

If your score on Problem Set 1 was less than 10, you may replace one of the three problems on this problem set by complete, correct solutions to all problems on Problem Set 1.